Dressing effects on elastic collisions in dusty plasmas

Young-Dae Jung*
Department of Physics, Hanyang University, Ansan, Kyunggi-Do 425-791, South Korea
and Department of Physics, 0319, University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093-0319

Sung-Soo Kim
Department of Mathematics, Hanyang University, Ansan, Kyunggi-Do 425-791, South Korea
(Received 11 February 2002; published 16 August 2002)

Elastic collisions between two dressed grains charged with the same sign in dusty plasmas are investigated using the first- and second-order eikonal method. An interaction potential model taking into account the cross terms of shield effects is applied to describe the interaction potential between dressed dust grains in dusty plasmas. The impact parameter method is applied to investigate the variation of the eikonal phase and elastic cross section as functions of dust charge, Debye length, and collision energy. The result shows that the potential well in the interaction potential plays an important role in the elastic cross section as well as in the eikonal phase. It is also found that the dressing effects significantly increase the elastic cross section and change the sign of the eikonal phase. It is also found that the second-order eikonal phase is caused by the pure plasma screening effects.

DOI: 10.1103/PhysRevE.66.026406 PACS number(s): 52.20.Hv, 52.27.Lw

I. INTRODUCTION

The elastic collision process has been known to be one of the major collision processes in many areas of physics. Recently, atomic collision processes in plasmas are of great interest since these processes can be used for plasma diagnostics [1–4]. The elastic electron-ion collision process [5] in dense plasmas have been investigated using the eikonal method [6,7]. The eikonal approximation has been widely used in many collision processes [5–7]. This eikonal method has a great advantage since we can obtain the general continuum wave functions for the collision system in terms of the eikonal phase including all physical information. Then, using the eikonal wave functions, we can readily calculate and investigate various cross sections for particle collisions in plasmas.

In recent years, there has been a considerable interest in the dynamics of gases and plasmas containing dust particles including collective effects, nonlinear effects, and strong electrostatic interaction between the charged components. These dust-plasma interactions occur not only in space plasmas but also in the laboratory plasmas. Several atomic processes in dusty plasmas have been investigated in order to obtain the information of relevant plasma parameters [8–10]. To the best of our knowledge, however, elastic collisions of two dressed grains charged with the same sign in dusty plasmas have not been investigated as yet. The theoretical calculation of collision cross sections in dust plasmas can be a useful tool for investigating interaction potentials in dusty plasmas. Thus, in this paper we investigate the elastic collision process between dressed dust grains charged with the same sign in dusty plasmas using the first- and second-order eikonal method. An interaction potential model [11] taking into account the cross terms of shield effects is applied to describe the interaction potential between negatively charged two dressed dust grains in dusty plasmas. The semiclassical impact parameter method is applied to visualize the first- and second-order eikonal phases as functions of the impact parameter and plasma parameters.

In Sec. II, we discuss the eikonal method and derive the first- and second-order eikonal phases and the elastic collision cross section. In Sec. III we obtain the analytic expressions of the eikonal phases and the elastic cross section for dressed grain-grain collisions in dusty plasmas including the dust-Debye sphere interaction terms, i.e., the dressing terms. We also investigate the dressing effects on the eikonal elastic collision cross section as well as on the total eikonal phase. Finally, in Sec. IV, a summary and discussion are given.

II. FIRST- AND SECOND-ORDER EIKONAL METHOD

Using the first-order theory arising from the integral Lippmann-Schwinger equation [12],

$$\Psi(\mathbf{r}) = \phi(\mathbf{r}) + \int d^3\mathbf{r}'\, G_0(\mathbf{r}-\mathbf{r}')\, U(\mathbf{r}')\, \Psi(\mathbf{r}'), \qquad (1)$$

where $U(\mathbf{r})$ $[\equiv 2\mu V(\mathbf{r})/\hbar^2]$ is the reduced potential energy, μ is the reduced mass of the collision system, $\Psi(\mathbf{r})$, $\phi(\mathbf{r})$, and $G_0(\mathbf{r}-\mathbf{r}')$ $[=-(4\pi)^{-1}e^{ik|\mathbf{r}-\mathbf{r}'|}/|\mathbf{r}-\mathbf{r}'|]$ are the solution of the time-independent Schrödinger equation, the solution of the homogeneous equation, the Green's function, i.e., the free-particle propagator, respectively, such that

$$(\nabla^2 + k^2)\Psi(\mathbf{r}) = U(\mathbf{r})\Psi(\mathbf{r}), \qquad (2)$$

$$(\nabla^2 + k^2)\phi(\mathbf{r}) = 0, \qquad (3)$$

$$(\nabla^2 + k^2)G_0(\mathbf{r}-\mathbf{r}') = \delta^3(\mathbf{r}-\mathbf{r}'), \qquad (4)$$

*Permanent address: Department of Physics, Hanyang University, Ansan, Kyunggi-Do 425-791, South Korea. Electronic address: yjung@bohr.hanyang.ac.kr; ydjung@physics.ucsd.edu

where $k\ (=\sqrt{2\mu E/\hbar^2})$ is the wave number, E is the collision energy, and $\delta^3(\mathbf{r}-\mathbf{r}')$ is the three-dimensional Dirac delta function. If we have chosen the z axis in the direction of the incident wave number k_i and using the expansion technique on the free outgoing Green's function [7], we then obtain the eikonal wave function $\Psi_E(\mathbf{r})$ as

$$\Psi_E(\mathbf{r})=(2\pi)^{-3/2}\exp\left[i\mathbf{k}_i\cdot\mathbf{r}-\frac{i}{2k_i}\int_{-\infty}^{z}dz'\,U(\mathbf{b},z')\right], \quad (5)$$

where $\mathbf{b}$ is the impact parameter and the integration in the phase factor is along a straight line parallel to $\mathbf{k}_i$. This eikonal method is equivalent to the Glauber approximation in the potential scattering. The relation between the Glauber method and other methods such as the Born and classical methods can be considered using the parameter $|V|d/\hbar v$ for the potential scattering, where $|V|$ is a typical strength of the interaction potential, d is the potential action distance, and v is the collision velocity. Since this parameter can be both greater and smaller than the unity in the Glauber approximation, the Glauber method has a wide range of applicability in the potential scattering process [13]. If we adopt a cylindrical coordinate system such that $\mathbf{r}=\mathbf{b}+z\hat{n}$, $\hat{\mathbf{n}}$ being a unit vector perpendicular to the momentum transfer $\mathbf{\Delta}\ (\equiv\mathbf{k}_i-\mathbf{k}_f)$, i.e., $\hat{\mathbf{n}}\cdot\mathbf{\Delta}=0$, and the integral over the variable z, the eikonal collision amplitude f_E is then found to be

$$f_E=-\frac{1}{4\pi}\int d^3\mathbf{r}\,e^{i\mathbf{\Delta}\cdot\mathbf{r}}U(\mathbf{r})\exp\left[-\frac{i}{2k_i}\int_{-\infty}^{z}dz'\,U(\mathbf{b},z')\right]. \quad (6)$$

After some algebra, the total elastic collision cross section using the eikonal method is given by

$$\sigma_E=\int d\Omega\,|f_E|^2,$$

$$=2\pi\int db\,b\left|\exp\left[i\left(\frac{1}{k}\chi_1(b)+\frac{1}{k^3}\chi_2(b)\right)\right]-1\right|^2, \quad (7)$$

where the differential solid angle $d\Omega$ was replaced by $d^2\mathbf{\Delta}/k^2$ (here, $|\mathbf{k}_i|=|\mathbf{k}_f|=k$) and $\chi_1(b)$ and $\chi_2(b)$ are the first-order and second-order eikonal phases, respectively,

$$\chi_1(b)=-\frac{\mu}{\hbar^2}\int_{-\infty}^{\infty}dz\quad V(r), \quad (8)$$

$$\chi_2(b)=-\left(\frac{\mu}{\hbar^2}\right)^2\int_{-\infty}^{\infty}dz\quad V(r)\left[V(r)+r\frac{d}{dr}V(r)\right] \quad (9)$$

with $r=(b^2+z^2)^{1/2}$.

III. DRESSED GRAIN-GRAIN COLLISION

Very recently, the interaction potential [11] between two dressed dust grains in dusty plasmas has been obtained including the interaction terms between two grains, two Debye spheres, and two grain-Debye sphere. We suppose that two dust grains possess spherical shape with radius a_1 and a_2, and charged negatively $Q_1\ (=-Z_1e)$ and $Q_2(=-Z_2e)$. Then, the total interaction potential energy between two dressed dust grains charged the same sign Q_1 and Q_2 can be obtained as

$$V(r)=\frac{Q_1Q_2}{r}e^{-r/\Lambda}\left(1-\frac{r}{2\Lambda}\right), \quad (10)$$

where Λ is the Debye length. This form of the interaction potential is valid for $\Lambda\gg a_1,a_2$. For typical laboratory and astrophysical dusty plasmas, it has been known that the Debye length is much greater than the grain size, i.e., $\Lambda\gg a$. Here, the pure exponential term $[(-Q_1Q_2/2\Lambda)e^{-r/\Lambda}]$ in Eq. (5) represents the attractive interactions between the grain-Debye sphere (two cross interactions). Recently, Chen *et al.* [14] used a more approximate method to calculate the interaction potential and then obtained a similar conclusion. If we neglect the interaction between the grain and the Debye sphere, the interaction potential $V(r)$ becomes the Yukawa type [15] potential $V(r)=Q_1Q_2e^{-r/\Lambda}/r$. The interaction potential Eq. (10) has a zero point at $r/\Lambda=2$ and a potential well near the minimum position $r/\Lambda\approx2.7$. Then, negatively charged two dust grains can attract each other beyond this minimum position. Various mechanisms of attractive effects for charged dust grains in dusty plasmas have been also proposed [16–18]. Recently, the horizontal part of the dust-dust interaction potential was measured for several plasma conditions [19]. The effective dusty charge and the screening length were also obtained. It has been known that the dynamic screening effect is identical to the static Debye screening since the plasma dielectric function becomes the static value when the velocity of the projectile is less than the thermal velocity of the plasma particles [20]. In dust-dust collisions, the velocity of the dust particle is usually less than the ion thermal velocity because of the large mass ratio M/m_i, where M is the dust mass and m_i is the ion mass. A discussion on the effects of the plasma ions in interactions of dust grain was given by a recent excellent paper by Vladimirov and Nambu [16]. However, we neglect the velocity effect on the interaction potential since it is small when the velocity of the projectile is less than the ion thermal velocity. Thus, the interaction potential Eq. (10) is quite reliable for describing dust-dust collision processes in dusty plasmas. The use of the interaction potential [Eq. (10)] and the impact parameter approach allows us to derive analytic expressions for the first-and second-eikonal phases as follows:

$$\frac{1}{k}\chi_1(\bar{b})=-\left(\frac{\mu}{m}\right)^{1/2}\frac{2Z_1Z_2}{\bar{E}^{1/2}}\int_0^{\infty}d\bar{z}\frac{\exp(-\sqrt{\bar{b}^2+\bar{z}^2})}{\sqrt{\bar{b}^2+\bar{z}^2}}\times\left(1-\frac{\sqrt{\bar{b}^2+\bar{z}^2}}{2}\right),$$

$$=-\left(\frac{\mu}{m}\right)^{1/2}\frac{2Z_1Z_2}{\bar{E}^{1/2}}\left[K_0(\bar{b})-\frac{\bar{b}}{2}K_1(\bar{b})\right], \quad (11)$$

TABLE I. Numerical values of the total elastic collision cross section in units of $\pi\Lambda^2$ for $Z_1=1000$, $Z_2=500$, $a_1=0.02$ μm, and $a_2=0.01$ μm.

$\bar{E}$	$\sigma_E(a_\Lambda,\bar{E})^a/\pi\Lambda^2$	$\sigma'_E(a_\Lambda,\bar{E})^b/\pi\Lambda^2$
100	1297.0	965.8
300	1183.6	931.7
600	1074.4	906.7

[a]The total elastic collision cross section $\sigma_E(a_\Lambda,\bar{E})$ including the interactions between the grain-Debye sphere.

[b]The total elastic collision cross section $\sigma'_E(a_\Lambda,\bar{E})$ neglecting the interactions between the grain-Debye sphere.

$$\frac{1}{k}\chi_2(\bar{b},a_\Lambda)=-\left(\frac{\mu}{m}\right)^{1/2}\frac{2(Z_1Z_2)^2}{\bar{E}^{3/2}}a_\Lambda\int_0^\infty d\bar{z}\,\frac{\exp(-2\sqrt{\bar{b}^2+\bar{z}^2})}{\sqrt{\bar{b}^2+\bar{z}^2}}\times\left(1-\frac{\sqrt{\bar{b}^2+\bar{z}^2}}{2}\right)\left(-\frac{3}{2}+\frac{\sqrt{\bar{b}^2+\bar{z}^2}}{2}\right)$$
$$=\left(\frac{\mu}{m}\right)^{1/2}\frac{2(Z_1Z_2)^2}{\bar{E}^{3/2}}a_\Lambda\left[\left(\frac{3}{2}+\frac{\bar{b}^2}{8}\right)K_0(2\bar{b})-\frac{5\bar{b}}{2}K_1(2\bar{b})+\frac{\bar{b}^2}{8}K_2(2\bar{b})\right], \tag{12}$$

where $\bar{b}$ $(\equiv b/\Lambda)$ is the scald impact parameter, $\bar{E}$ $(\equiv E/R=\mu v^2/2R)$ is the scaled collision energy, μ $[=M_1M_2/(M_1+M_2)]$ is the reduced mass of two dust grains with mass M_1 and M_2, R $(=me^4/2\hbar^2\cong 13.6$ eV) is the Rydberg constant, m is the electron mass, a_Λ $(\equiv a_0/\Lambda)$ is the scaled reciprocal Debye length, a_0 $(=\hbar^2/me^2)$ is the Bohr radius, and K_n are the modified Bessel functions [21] with order n. It is important to note that the second-order eikonal phase $\chi_2(\bar{b},a_\Lambda)$ vanishes when $\Lambda\to\infty$. Thus, it is found that the second-order eikonal phase is the pure plasma screening effect. Then, the scaled total eikonal elastic dust-dust collision cross section in units of $\pi\Lambda^2$ is given by

$$\sigma_E(a_\Lambda,\bar{E})/\pi\Lambda^2=\int d\bar{b}\,2\bar{b}|\exp[i\chi(\bar{b},a_\Lambda,\bar{E})]-1|^2, \tag{13}$$

where $\chi(\bar{b},a_\Lambda,\bar{E})$ $[\equiv\chi_1(\bar{b})/k+\chi_2(\bar{b},a_\Lambda)/k^3]$ is the total eikonal phase. In this work, we neglect the charging effect [22] that is a defining characteristic of dusty plasmas. It turns out that mutual charging effects are found to be unimportant at the grain-grain collision considered in this work, mainly one Debye length. However, the charging effects start to become important at distances of the closest approach smaller than the Debye length. The Debye shield distortions start to become a concern at small separations. However, we are interested in distant collisions $(b\geqslant\Lambda)$ between two dressed dust grains. Thus, the Debye shield distortion effects, i.e., nonlinear screening effect on the interaction potential is expected to be quite small and then negligible for distant encounters $(b\geqslant\Lambda)$ of dust grains. Since the results, Eqs. (11), (12), and (13), are all scaled in units of the Debye length, these results can be applied to duty plasmas where the Debye screening effect plays an important role. Thus, the results of the eikonal phases Eqs. (11) and (12) and the collision cross section [Eq. (13)] are found to be quite reliable for the domain $b\geqslant\Lambda$. If we neglect the grain-Debye sphere interaction terms, i.e., the dressing terms, the first- and second-eikonal phases are just found to be

$$\frac{1}{k}\chi'_1(\bar{b})=-\left(\frac{\mu}{m}\right)^{1/2}\frac{2Z_1Z_2}{\bar{E}^{1/2}}K_0(\bar{b}), \tag{14}$$

$$\frac{1}{k}\chi'_2(\bar{b},a_\Lambda)=\left(\frac{\mu}{m}\right)^{1/2}\frac{2(Z_1Z_2)^2}{\bar{E}^{3/2}}a_\Lambda K_0(2\bar{b}). \tag{15}$$

In order to explicitly investigate the dressing effects, i.e., the grain-Debye sphere interaction effects, on the eikonal elastic cross section for dressed dust-dust collisions in dusty plasmas, specifically, we set $a_1=0.02$ μm, $a_2=0.01$ μm,

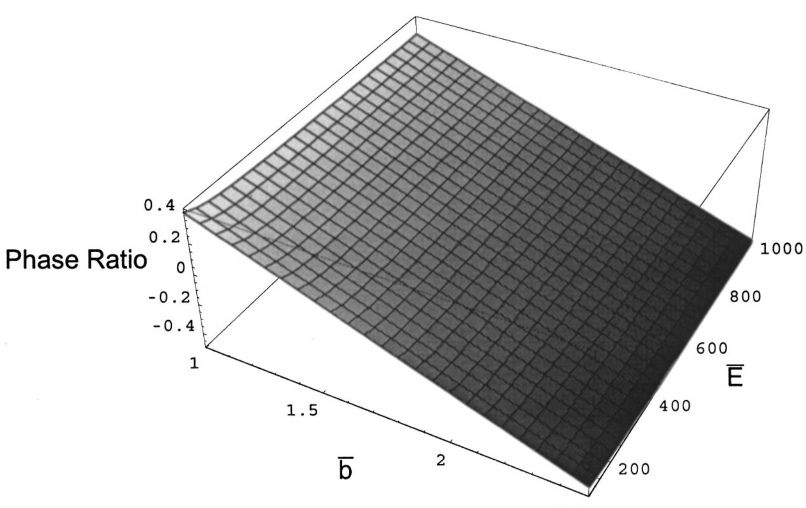

FIG. 1. The three-dimensional plot of the ratio of the total eikonal phase $\chi(\bar{b},a_\Lambda,\bar{E})$ including the interactions between the grain-Debye sphere [Eqs. (11) and (12)] to the total eikonal phase $\chi'(\bar{b},a_\Lambda,\bar{E})$ neglecting the interactions between the grain-Debye sphere [Eqs. (14) and (15)] as a function of the scaled collision energy and the scaled impact parameter for $\bar{b}\geqslant 1$.

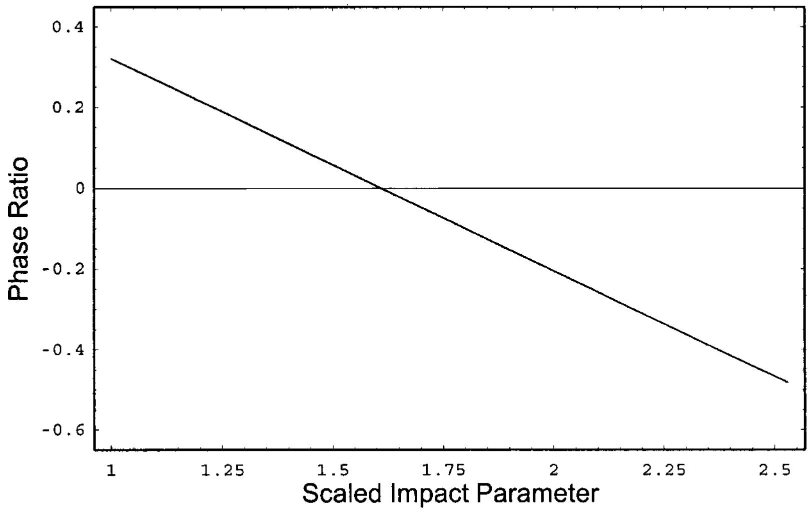

FIG. 2. The ratio of the total eikonal phase $\chi(\bar{b},a_\Lambda,\bar{E})$ including the interactions between the grain-Debye sphere to the total eikonal phase $\chi'(\bar{b},a_\Lambda,\bar{E})$ neglecting the interactions between the grain-Debye sphere as a function of the scaled impact parameter when $\bar{E}=300$.

$Z_1=1000$, $Z_2=500$, $\Lambda=50a_1$, and the density of the dust grains is $\rho\cong 2\ \mathrm{g\,cm^{-3}}$ [23]. Since we choose $\Lambda \gg a_1,\ a_2$, the form of the interaction potential [Eq. (10)] is reliable to describe the interaction between two dressed dust grains. Table I shows numerical values of the total elastic collision cross section in units of $\pi\Lambda^2$. As we see in this table, the dressing effects significantly increase the elastic cross section, for example, 26% and 21% for $\bar{E}=100$ and $\bar{E}=300$. It is found that the dressing effects are reduced as an increase of the collision energy. Figure 1 shows the three-dimensional plot of the ratio of the total eikonal phase $\chi(\bar{b},a_\Lambda,\bar{E})$ including the interactions between the grain-Debye sphere [Eqs. (11) and (12)] to the total eikonal phase $\chi'(\bar{b},a_\Lambda,\bar{E})$ neglecting the interactions between the grain-Debye sphere [Eqs. (14) and (15)] as a function of the scaled collision energy and the scaled impact parameter for $\bar{b}\geqslant 1$ since our results Eqs. (11) and (12) are quite reliable for $b\geqslant\Lambda$. Figure 2 shows the ratio of the total eikonal phase $\chi(\bar{b},a_\Lambda,\bar{E})$ including the interactions between the grain-Debye sphere to the total eikonal phase $\chi'(\bar{b},a_\Lambda,\bar{E})$ neglecting the interactions between the grain-Debye sphere when $\bar{E}=300$. As we see in these figures, it is found that the dressing effects change the sign of the eikonal phase for $\bar{b}>1.6$. It means that the attractive interaction is dominant for long-range collisions. This attractive behavior of the dust particles is a result of the plasma polarization. The position of the potential well due to the attractive interaction caused by the polarization does not depend on the charges of dust grains for $\Lambda\gg a_1,\ a_2$. Very, recently Ivanov [24] obtained this polarization interaction in dusty plasmas and got the same result with Eq. (10). Thus, the charging effect of dust particles is found to be unimportant in this case since the position of the equilibrium between the repulsive and attractive interactions is greater than the Debye length. The result shows that the potential well produced by the attractive interaction between dressed dust-dust grains plays an important role in collision processes in dusty plasmas.

IV. SUMMARY AND DISCUSSION

We investigate the elastic collision process between two dressed grains charged with the same sign in dusty plasmas using the eikonal method. An interaction potential model taking into account the cross terms of shield effects is applied to describe the interaction potential between dressed dust grains in dusty plasmas. The impact parameter method is applied to investigate the variation of the first- and second-eikonal eikonal phases and the elastic cross section as functions of dust charge, Debye length, and collision energy. The result shows that the potential well caused by the dressing effect in the interaction potential plays a significant role in the elastic collision cross section as well as in the eikonal phase. It is also found that the dressing effects significantly increase the elastic collision cross section. The dressing effects are also found to be decreased with increasing the collision energy. It is found that the dressing effects also change the sign of the eikonal phase for $\bar{b}>1.5$ due to the long-range dust-dust interaction. It should be noted that the second-order eikonal phase is caused by the pure plasma screening effects. These results will provide useful information on the collision processes between two dressed grains charged with the same sign in dusty plasmas.

ACKNOWLEDGMENTS

One of the authors (Y.-D.J.) gratefully acknowledges Dr. M. Rosenberg and Professor D. A. Mendis for useful discussions and comments on dusty plasmas. He also thanks Professor R. J. Gould and Professor T. M. O'Neil for warm hospitality while visiting the University of California, San Diego. This work was supported by Grant No. R02-2002-000-00025-0 from the Basic Research Program of the Korean Science and Engineering Foundation.

[1] V. N. Tsytovich, *Lectures on Non-Linear Plasma Kinetics* (Springer-Verlag, Berlin, 1997).
[2] Y.-D. Jung, Phys. Plasmas **2**, 1775 (1995).
[3] V. S. Lebedev and I. L. Beigman, *Physics of Highly Excited Atoms and Ions* (Springer-Verlag, Berlin, 1997).
[4] Y.-D. Jung, Phys. Lett. A **284**, 275 (2001).
[5] I. H. Hutchinson, *Principles of Plasma Diagnostics* (Cambridge University, New York, 1987).
[6] C. J. Joachain, *Quantum Collision Theory*, 3rd ed. (North-Holland, Amsterdam, 1983).
[7] P. G. Burke and C. J. Joachain, *Theory of Electron-Atom Collisions, Part 1: Potential Scattering* (Plenum, New York, 1995).
[8] V. N. Tsytovich, Comments Plasma Phys. Controlled Fusion **15**, 349 (1994).
[9] V. N. Tsytovich, Ya. K. Khodataev, and R. Bingham, Comments Plasma Phys. Controlled Fusion **17**, 249 (1996).
[10] P. Bliokh, V. Sinitsin, and V. Yaroshenko, *Dusty and Self-Gravitational Plasma in Space* (Kluwer, Dordrecht, 1995).
[11] L. Wang, Comments Mod. Phys., Part C **1**, 117 (1999).
[12] A. Gonis and W. H. Butler, *Multiple Scattering in Solids* (Springer-Verlag, New York, 2000).
[13] G. A. Pavlov, *Transport Processes in Plasmas with Strong Coulomb Interaction* (Gordon and Breach, London, 2000)
[14] Y.-P. Chen, H. Luo, M.-F. Ye, and M. Y. Yu, Phys. Plasmas **6**, 699 (1999).
[15] H. Ikezi, Phys. Fluids **29**, 1764 (1986).
[16] S. V. Vladimirov and M. Nambo, Phys. Rev. E **52**, 2172 (1995).
[17] M. Nambo, S. V. Vladimirov, and P. K. Shukla, Phys. Lett. A **203**, 40 (1995).
[18] Ya. K. Khodataev, G. E. Morfill, and V. N. Tsytovich, J. Plasma Phys. **65**, 257 (2001).
[19] U. Konopka, G. E. Mrofill, and L. Patke, Phys. Rev. Lett. **84**, 891 (2000).
[20] Y.-D. Jung, Phys. Plasmas **4**, 21 (1997).
[21] C. M. Bender and S. A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers* (Springer, New York, 1999).
[22] D. A. Mendis and M. Rosenberg, Annu. Rev. Astron. Astrophys. **32**, 419 (1994).
[23] F. Verheest, *Waves in Dusty Space Plasmas* (Kluwer, Dordrecht, 2000)
[24] A. S. Ivanov, Phys. Lett. A **290**, 304 (2001).